$\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{2}$

"there exists"

1) let $\sqrt{2}$ be a rational no, then $\exists$ integers p, q such that $\sqrt{2} = \frac{p}{q}$ and $(p,q) = 1$

$\Rightarrow$ $p = \sqrt{2}\,q$ squaring both sides we get

$p^2 = 2q^2$ (1) since p, q are integers

$\Rightarrow$ p^2 is a $m(2)$ hence p is a $m(2)$

$\because$ p is a $m(2)$ $\Rightarrow$ $\exists$ a integer k such that $p = 2k$

Substituting in (1) we get

$p^2 = 4k^2 = 2q^2 \Rightarrow q^2 = 2k^2$

$\Rightarrow$ q is a $m(2)$ but this contradicts our initial hypothesis of $(p,q) = 1$ because $(p,q) = 2$ by our assumpt

Hence our initial assumption that $\sqrt{2}$ is rational is incorrect and hence contradiction we say $\sqrt{2}$ is irrational

$\sqrt[3]{2}$ as irrational.

① $n \in \mathbb{Z}$, if n^2 is odd — then n is also odd | but n is even

n^2 is odd then on division by 2 it leave a remainder of ('1')

but n is even so let $\boxed{n = 2k}$ k is some Integer then $\boxed{n^2 = 4k^2}$ ✓

as k is an integer ⇒ n^2 is divisible by 2 Hence n^2 is not leaving a remainder of 1 and hence must be even. But contradicts the fact that n^2 is odd. Hence n cannot be even n^2 is odd. Hence n is odd

n^2 is odd then n must be odd

n^2 as $\boxed{\text{even}}$ ← even

$a, b \in \mathbb{Z}$ $\quad 12a^2 - 6b^2 \cancel{=} 0 \quad 12a^2 - 6b^2 \neq 0$

$12a^2 \cancel{-} 6b^2 = 0$

$\boxed{b} = \sqrt{2}a$ ✓ $\quad 2a^2 \cancel{=} b^2$

$\boxed{\sqrt{2}} \cancel{=} \boxed{\frac{b}{a}}$ $\quad b, a \in \mathbb{Z}$ Rational.

$\sqrt{2}\,7$ is not rational

Sherlock Homes $\rightarrow$ Eliminate $\boxed{\text{Impossible}}$ $\left\{ \boxed{\text{possible}} \right\}$

$|2a^2 - 6b^2 \neq 0|$

$m^2 - n^2 = 1$ ✓

$(m-n)(m+n) = 1 \times 1$

m, n are positive integer

$\left.\begin{array}{l} m-n = 1 \\ m+n = 1 \end{array}\right\}$ $2m = 2$

$m = 1$

$\boxed{n=0}$

$\boxed{m, n \geq 1}$

$n = 0$

7 $\log_2 3 = \dfrac{p}{q}$ ✓ Mistake

$(p, q) = 1$

$\Rightarrow (2^{p/q})^q = 3^q$

$2^0 \quad \underbrace{2^p}_{} = 3^q \quad 3^0$

p, q are both '0'

const

$\{ \text{even} \neq \text{odd} \} =$

Portia

Malvolio

breaches

$$x^3 + x - 1 = 0$$

Irrat $x = p/q$

$$\frac{p^3}{q^3} + \frac{p}{q} - 1 = 0$$

$$p^3 + pq^2 - q^3 = 0$$

$$(p, q) = 1$$

$$p^3 = q^3 - pq^2$$

$$p^3 = \underline{q(q^2 - pq^2)}$$

$q \to p$ by q,

p, q $(p, q) \neq 1$